Elements of Mass Transfer

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mass transfer: — molecular processes
— turbulent processes

molecular	turbulent
ideal gas collisions	fluid flow
slow	depends on velocity of eddy
small spacial scales	depend on velocity & size of eddy

mass transfer rate laws:

— non-reactive gas mixture
— binary mixture of A & B

Fick's law of Diffusion: describes the rate at which one species moves through the other

1-D binary diffusion

$$\dot{m}_A'' = Y_A(\dot{m}_A'' + \dot{m}_B'') - \rho D_{AB}\frac{dY_A}{dx}$$

mass flow of species A per unit area

mass flow of species A associated w/ bulk flow per unit area

mass flow of species A assoc. w/ molecular diffusion per unit area

$\dot{m}_A''$ = mass flux of species A

Y_A = Mass fraction of species A

Mass flux is defined as the mass flow rate of species A per unit area perpendicular to the flow.

$$\dot{m}_A'' = \dot{m}_A / \text{Area}$$

Binary Diffusivity (D_{AB}) is a property of a mixture and has units $[m^2/s]$

$$\dot{m}_A'' = Y_A(\dot{m}_A'' + \dot{m}_B'') - \cancelto{0}{\rho D_{AB} \frac{dY_A}{dx}}$$

$$\dot{m}_A'' = Y_A(\dot{m}''_{total}) \equiv \text{Bulk flow of A}$$

$$\text{Diffusional flux} \equiv -\rho D_{AB} \frac{\partial Y_A}{\partial x}$$

Multi-directional System:

$$\dot{m}_A'' = Y_A(\dot{m}_A'' + \dot{m}_B'') - \rho D_{AB} \nabla Y_A$$

$\dot{m}_A''$ & $\dot{m}_B''$ $\Rightarrow$ vector quantities

Molar quantities

$$\dot{N}_A'' = X_A(\dot{N}_A'' + \dot{N}_B'') - c D_{AB} \nabla X_A$$

$\hookrightarrow$ $\dot{N}_A''$ = molar flux $(\text{kmol}/\text{s-m}^2)$

c = conc. of mixture

$$\dot{m}'' = \dot{m}_A'' + \dot{m}_B''$$

mixture mass flux   species A mass flux   species B mass flux

1-D:

$$\dot{m}'' = Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x} + Y_B \dot{m}'' - \rho D_{AB} \frac{\partial Y_B}{\partial x}$$

$$\dot{m}'' = \underbrace{(Y_A + Y_B)}_{1} \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x} - \rho D_{AB} \frac{\partial Y_B}{\partial x}$$

$$\underbrace{- \rho D_{AB} \frac{\partial Y_A}{\partial x}}_{\text{diffusional flux A}} - \underbrace{\rho D_{AB} \frac{\partial Y_B}{\partial x}}_{\text{diffusional flux of B}} = 0$$

in general $$\sum_i \dot{m}''_{i,diff} = 0$$

- Diffusion as a result of conc. gradients is called ordinary diffusion

- gradients of temperature (Soret effect) ⇐ temperature diffusion
- gradients of pressure ← pressure diffusion

Species conservation

mixture mass flux | species A mass flux | species B mass flux

1-D:

$$\dot{m}'' = Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x} + Y_B \dot{m}'' - \rho D_{AB} \frac{\partial Y_B}{\partial x}$$

$$\dot{m}'' = \cancel{(Y_A + Y_B)}^{1} \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x} - \rho D_{AB} \frac{\partial Y_B}{\partial x}$$

$$\underbrace{- \rho D_{AB} \frac{\partial Y_A}{\partial x}}_{\text{diffusional flux A}} - \underbrace{\rho D_{AB} \frac{\partial Y_B}{\partial x}}_{\text{diffusional flux of B}} = 0$$

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<u>Species conservation</u>

per unit volume $(kg/m^3\text{-}s)$

if $m_{A,cv} = Y_A \, m_{cv} = Y_A \, \rho V_{cv} = \underline{Y_A \rho A \Delta x}$

$$\frac{d m_A}{\partial t} = [\dot{m}_A'' A]_x - [\dot{m}_A'' A]_{x+\Delta x} + \dot{m}_A''' V$$

$$A \Delta x \, \frac{d(\rho Y_A)}{\partial t} = A\left[Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}\right]_x - A\left[Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}\right]_{x+\Delta x} + \dot{m}_A''' A \Delta x$$

divide thru by $A \Delta x$

$\Delta x \to 0$

$$\Rightarrow \frac{\partial(\rho Y_A)}{\partial t} = -\frac{\partial}{\partial x}\left[Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}\right] + \dot{m}_A'''$$

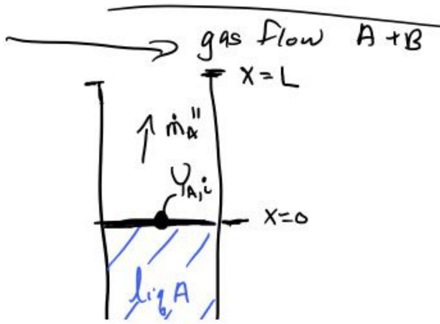
$\Rightarrow$ steady flow

$$\dot{m}_A''' - \frac{\partial}{\partial x}\left[Y_A \dot{m}'' - \rho D_{AB} \frac{d Y_A}{\partial x}\right] = 0$$

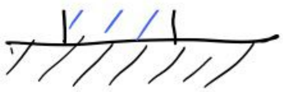
$\Rightarrow$ multi-dimensional

$$\dot{m}_A''' - \nabla \cdot \dot{m}_A'' = 0$$

The Stefan Problem



- $Y_{A,i} > Y_A$ flowing gas
- Steady state
- B ... insoluble in A



- B is insoluble in A
- Stagnant layer of B in column.

Cons. of mass:

$$\dot{m}''(x) = \text{const.} = \dot{m}''_A + \dot{m}''_B$$

$$\dot{m}''_B = 0 \rightarrow \dot{m}''_A = \text{const} = \dot{m}''(x)$$

1-D consv. of species:

$$\dot{m}_A'' = Y_A \dot{m}_A'' - \rho D_{AB} \frac{dY_A}{dx}$$

$$\frac{\dot{m}_A'' dx}{\rho D_{AB}} = \frac{Y_A \dot{m}_A'' dx}{\rho D_{AB}} - dY_A$$

$$dY_A = \frac{-\dot{m}_A''}{\rho D_{AB}} dx + \frac{Y_A \dot{m}_A''}{\rho D_{AB}} dx$$

$$dY_A = \frac{-\dot{m}_A''}{\rho D_{AB}} dx\,(1 - Y_A)$$

$$\frac{dY_A}{1 - Y_A} = \frac{-\dot{m}_A''}{\rho D_{AB}} dx$$

$$-\frac{\dot{m}_A''}{\rho D_{AB}} x = -\ln(1 - Y_A) + C$$

BC: $x = 0 \Rightarrow Y_A = Y_{A,i}$

$$-\frac{\dot{m}_A''}{\rho D_{AB}}(0) = -\ln(1 - Y_{A,i}) + C$$

$$\hookrightarrow C = \ln(1 - Y_{A,i})$$

$$\rightarrow \frac{+\dot{m}_A''}{\rho D_{AB}} x = +\ln(1-Y_A) \mp \ln(1-Y_{A,i})$$

$$\frac{\dot{m}_A''}{\rho D_{AB}} x = \ln\left(\frac{1-Y_A}{1-Y_{A,i}}\right)$$

$$\exp\left(\frac{\dot{m}_A' x}{\rho D_{AB}}\right) = \frac{1-Y_A}{1-Y_{A,i}}$$

$$\Rightarrow Y_A(x) = 1-(1-Y_{A,i})\exp\left(\frac{\dot{m}_A'' x}{\rho D_{AB}}\right)$$

B.C. @ $x=L \rightarrow Y_A = Y_{A,\infty}$

$$Y_{A,\infty} = 1-(1-Y_{A,i})\exp\left(\frac{\dot{m}_A'' L}{\rho D_{AB}}\right)$$

$$\frac{(1-Y_{A,\infty})}{(1-Y_{A,i})} = \exp\left(\frac{\dot{m}_A'' L}{\rho D_{AB}}\right)$$

$$\hookrightarrow \dot{m}_A'' = \frac{\rho D_{AB}}{L}\ln\left(\frac{1-Y_{A,\infty}}{1-Y_{A,i}}\right)$$

— Equilibrium : I.G.

$$\hookrightarrow P_{A,i} = P_{sat}(T_{liq,i})$$

$$\hookrightarrow X_{A,i} = \frac{P_{A,i}}{P} = \frac{P_{sat}(T_{liq,i})}{P}$$

$$Y_{A,i} = \frac{P_{sat}(T_{liq,i})}{P} \frac{MW_A}{MW_{mix,i}}$$