# One film burning rate

Tuesday, November 28, 2017 12:14 PM

Estimate the burning rate of a 250 micrometer diameter carbon particle burning in still air ($Y_{O2,inf}$ = 0.233) at 1 atm. The particle temperature is 1800 K, and the kinetic rate constant $k_c$ is 13.9 m/s. Assume the mean molecular weight of the gases at the surface is 30 kg/kmol. Also, what combustion regime prevails?

P = 1 atm = 101325 Pa
$R_u$ = 8315 Pa m$^3$ / mol - K
D = 1.6 x 10$^{-5}$ m$^2$/s @ 393 K.

$$R_{diff} = \frac{\nu_1 + Y_{O_2,s}}{\rho D 4 \pi r_s}$$

$$R_{diff} \doteq 5.41 \times 10^{7} \ s/kg$$

$\nu_1 = 2.664$

$Y_{O_2 s} \approx 0$

$$D_{1800K} = \left(\frac{1800K}{393\ K}\right)^{1.5} \left(1.6 \times 10^{-5} \frac{m^2}{s}\right)$$

$$= 1.57 \times 10^{-4} \ m^2/s$$

$$\rho = \frac{P}{(R_u/mw_{mix}) T_s}$$

$$= \frac{1atm \ (30 \ kg/kmol)}{(0.082057 \frac{m^3 atm}{kmol-K}) (1800K)}$$

$$= 0.20 \ kg/m^3$$

$r_s = 125 \times 10^{-6}$ m

$$R_{kin} = \nu_1 R_u T_s \left(\frac{mw_{O_2}}{mw_c}\right)$$

$$\frac{\quad}{4\pi r_s^2 MW_{mix} k_c P}$$

$$= 4.81 \times 10^{6} \ s/kg$$

$$\dot{m}_c = \frac{Y_{O_2,\infty} - 0}{R_{kin} + R_{diff}} = \frac{0.233}{4.81E6 + 5.41E7}$$

$$= 3.96E-9 \ kg/s$$

$$Y_{O_2,s} - 0 = \dot{m}_c R_{kin}$$

$$= 3.96E-9 \ (4.81 E6)$$

$$= 0.019 \quad 1.9\%$$

— Repeat to convergence

# One film temperature problem

Tuesday, November 28, 2017 2:34 PM

In the combustion of solid fuels, radiation usually plays a key role. Estimate the gas temperature required to keep a 250 micrometer diameter burning carbon particle at 1800 K for (1) when there is no radiation ($T_s = T_{surr}$) and (2) when the particle radiates as a blackbody to surroundings at 300 K. Conditions are identical to the earlier problem.

P = 1 atm = 101325 Pa
$R_u$ = 8315 Pa $m^3$ / mol - K
D = 1.57 x $10^{-4}$ $m^2/s$ @ 1800 K
$c_{pg}$ (1800 K) = 1286 J/kg-K
$k_g$ (1800 K) = 0.12 W/m-K
$m_c$ = 3.96 x $10^{-9}$ kg/s
$\Delta h_c$ = 3.2765 x $10^7$ J/kg
$\varepsilon$(blackbody) = 1
$\sigma$ = 5.67 x $10^{-8}$ W/$m^2K^4$ (Stefan-Boltzmann constant)

i) no radiation

$$\dot{m}_c \Delta h_c = \dot{m}_c c_{pg} \left[ \frac{\exp\left(\frac{-\dot{m}_c c_{pg}}{4\pi k_g r_s}\right)}{1 - \exp(-\dot{m}_c c_{pg}/4\pi k_g r_s)} \right] (T_s - T_\infty)$$

$$\frac{\Delta h_c}{c_{pg}} \left[ \frac{1 - \exp(\ )}{\exp(\ )} \right] = T_s - T_\infty$$

$$T_\infty = 1102\ K$$

ii)

$$\dot{Q}_{rad} = \varepsilon_s 4\pi r_s^2 \sigma (T_s^4 - T_{surr}^4)$$
$$= (1)\, 4\pi (125\times10^{-6})^2 (5.67E{-8}) (1800^4 - 300^4)$$
$$= 0.1168 W$$

$$\frac{\dot{m}_c \Delta h_c - \dot{Q}_{rad}}{\dot{m}_c c_{pg}} = \left[ \frac{\exp(\ )}{1 - \exp(\ )} \right] (T_s - T_\infty)$$

$$T_\infty = 1730\ K$$

# 1 & 2 Film burning rate problem

Tuesday, November 28, 2017 2:40 PM

**Assuming diffusion-controlled combustion and identical conditions ($Y_{O2,inf}$ = 0.233), compare the burning rates predicted by the one and two film models.**

$$\dot{m}_C = 4\pi r_s \rho D \ln(1+B)$$

for 1-film: $B = B_{O_2}$

2-film: $B = B_{CO_2}$

$$\frac{\dot{m}_C(\text{2-film})}{\dot{m}_C(\text{1-film})} = \frac{\cancel{4\pi r_s \rho D} \ln(1+B_{CO_2})}{\cancel{4\pi r_s \rho D} \ln(1+B_{O_2})} = \frac{\ln(1+B_{CO_2})}{\ln(1+B_{O_2})}$$

$$B_{CO_2} = \frac{2Y_{O_2,\infty} - \cancel{[(\nu_s-1)/\nu_s]\, Y_{CO_2,s}}}{\nu_s - 1 + \cancel{[(\nu_s-1)/\nu_s]\, Y_{CO_2,s}}}$$

$Y_{O_2,\infty} = 0.233$

$\nu_s = 3.664$

$Y_{CO_2,s} = 0$

$$B_{CO_2} = \frac{2Y_{O_2,\infty}}{\nu_s - 1} = \frac{2(0.233)}{3.664-1} = 0.175$$

$$B_{O_2} = \frac{Y_{O_2,\infty} - Y_{O_2,s}}{\nu_1 + Y_{O_2,s}} = \frac{0.233 - 0}{2.664 + 0} = 0.0875$$

$$\frac{\dot{m}_{C_2}}{\dot{m}_{C_1}} = \frac{\ln(1+0.175)}{\ln(1+0.0875)} = 1.92$$

* 1-film $C \rightarrow CO_2$

2-film $C \rightarrow CO$

$$2C + 1O_2 \rightarrow 2CO$$

$$\nu_1 = \frac{MW_{O_2}}{2MW_C} = \frac{31.999 \text{ kg } O_2}{24.02 \text{ kg } C} = 1.333$$

$$B_{O_2} = \frac{Y_{O_2,\infty} - \cancel{Y_{O_2,s}}^{\,0}}{\nu_1 + \cancel{Y_{O_2,s}}^{\,0}} = \frac{0.233}{1.333} = 0.175$$

# 2 film burning rate problem

Tuesday, November 28, 2017 2:42 PM

**Use the two film model to estimate the burning rate of a 70 μm diameter carbon particle burning in air ($Y_{O2,inf}$ = 0.233). The surface temperature is 1800 K, and the pressure is 1 atm. Assume the molecular weight of the gaseous mixture at the particle surface is 30 kg/kmol.**

P = 1 atm = 101325 Pa

$R_u$ = 8315 Pa m³ / mol - K

D = 1.57 x $10^{-4}$ m²/s @ 1800 K.

$\rho$ = 0.2 kg/m³

$MW_C$ = 12.01 kg/kmol

$MW_{CO2}$ = 44.01 kg/kmol

$$k_c = 4.016 \times 10^8 \exp\left(\frac{-29790}{1800}\right) = 26.07 \text{ m/s}$$

Surface kinetics:

$$\dot{m}_c = 4\pi r_s^2 k_c \frac{MW_C \, MW_{mix}}{MW_{O2}} \frac{P}{R_u T_s} Y_{O_2,s}$$

$$= 4\pi (35 \times 10^{-6} m)^2 (26.07 \tfrac{m}{s}) \frac{(12.01)(30)}{(44.01)} \frac{101325}{(8315)(1800)} Y_{O_2,s}$$

$$= 2.22 \times 10^{-8} Y_{O_2,s} \quad (kg/s)$$

Closure

$$\dot{m}_c = 4\pi r_s \rho D \ln(1 + B_{CO_2})$$

$$= 4\pi (35 \times 10^{-6})(0.2)(1.57 \times 10^{-4}) \ln(1 + B_{CO_2})$$

$$= 1.381 \times 10^{-8} \ln(1 + B_{CO_2})$$

$$B_{CO_2} = \frac{2 Y_{O_2,\infty} - [(\nu_s - 1)/\nu_s] Y_{CO_2,s}}{(\nu_s - 1) + [(\nu_s - 1)/\nu_s] Y_{CO_2,s}}$$

$$= \frac{2(0.233) - [(3.664-1)/3.664] Y_{CO_2,s}}{(3.664-1) + [(3.664-1)/3.664] Y_{CO_2,s}}$$

$$= \frac{0.466 - 0.727 Y_{CO_2,s}}{2.664 - 0.727 Y_{CO_2,s}}$$

$$\dot{m}_c = 1.9 \times 10^{-9} \text{ kg/s}$$

# Burning time problem

Tuesday, November 28, 2017 2:49 PM

**Estimate the lifetime of a 70 μm diameter carbon particle assuming diffusionally controlled combustion at the conditions given for the previous problems. Assume the carbon density is 1900 kg/m³**

**$P = 1$ atm = 101325 Pa**
**$R_u = 8315$ Pa m³ / mol - K**
**$D = 1.57 \times 10^{-4}$ m²/s @ 1800 K.**
**$\rho = 0.2$ kg/m³**

$$t_c = D_0^2 / K_B$$

$$K_B = \frac{8 \rho D_{AB}}{\rho_C} \ln(1 + B_{CO_2})$$

$$= \frac{8 (0.20)(1.57 \times 10^{-4})}{1900} \ln(1 + 0.175)$$

$$= 2.13 \times 10^{-8} \ m^2/s$$

$$t_c = (70 \times 10^{-6} m)^2 / 2.13 \times 10^{-8} \ m^2/s$$

$$\boxed{= 0.23 \ s}$$