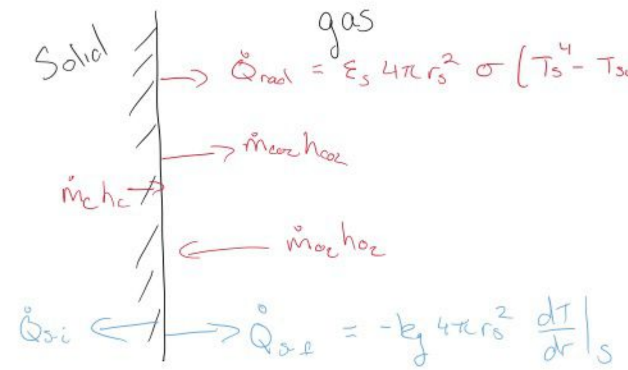
in between ~ 1 $\dot{m}_c = Y_{O_2,\infty} / (R_{kin} + R_{diff})$

Energy Conservation



$$\underbrace{\dot{m}_c h_c + \dot{m}_{O_2} h_{O_2} - \dot{m}_{CO_2} h_{CO_2}}_{\dot{m}_c h_{comb}} = \cancel{\dot{Q}_{s-i}}^{\,0} + \dot{Q}_{s-f} + \dot{Q}_{rad}$$

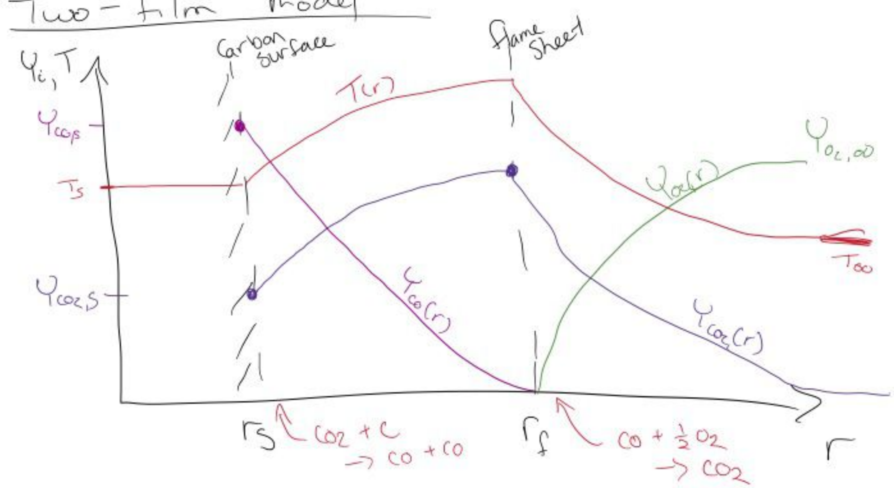
$$\boxed{\dot{m}_c h_{comb} = -k_g 4\pi r_s^2 \left.\frac{dT}{dr}\right|_s + \varepsilon_s 4\pi r_s^2 \sigma (T_s^4 - T_{surr}^4)}$$

w/ a droplet model:

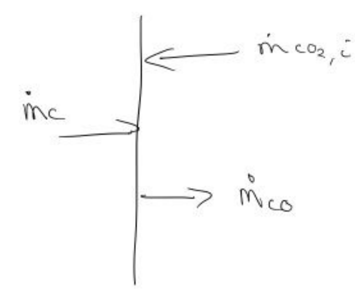
$$\left.\frac{dT}{dr}\right|_{r_s} = \frac{(c_p/4\pi k_g)\dot{m}_c}{r_s^2}\left[\frac{(T_\infty - T_s)\exp\left(\frac{-c_p \dot{m}_c}{4\pi r_s k_g}\right)}{1-\exp\left(-c_p \dot{m}_c/4\pi r_s k_g\right)}\right]$$

$$\Rightarrow \dot{m}_c \Delta h_{comb} = \dot{m}_c c_{pg}\left[\frac{\exp\left(\frac{-\dot{m}_c c_{pg}}{4\pi r_s k_g}\right)}{1-\exp\left(\frac{-\dot{m}_c c_{pg}}{4\pi r_s k_g}\right)}\right](T_s - T_\infty)$$
$$+ \varepsilon_s 4\pi r_s^2 \sigma (T_s^4 - T_{surr}^4)$$

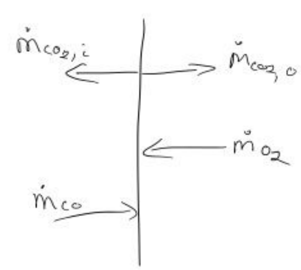
Two-film model



Surface



flame sheet



$\Big|_{r=r_s}$ $\Big|_{r=r_f}$

at surface:

$$\dot{m}_C + \dot{m}_{CO_2,i} = \dot{m}_{CO}$$

$$1\ kg\ C + \nu_s\ kg\ CO_2 = (1+\nu_s)\ kg\ CO$$

at flame sheet:

$$\dot{m}_{CO} + \dot{m}_{O_2} = \dot{m}_{CO_2,i} + \dot{m}_{CO_2,o}$$

$$(\dot{m}_{CO} - \dot{m}_{CO_2,i}) + \dot{m}_{O_2} = \dot{m}_{CO_2,o}$$

$$\dot{m}_C + \dot{m}_{O_2} = \dot{m}_{CO_2,o}$$

$$1 kg\ C + \nu_f\ kg\ \dot{m}_{O_2} = (1+\nu_f)\ kg\ CO_2$$

$$\nu_s = \frac{44.01}{12.01} = 3.664$$

$$\nu_f = \nu_s - 1 = 2.664$$

$MW_{CO_2} = MW_C + MW_{O_2}$

$$\nu_f = \frac{MW_{CO_2} - MW_C}{MW_C} = \nu_s - 1$$

$\dot{m}_{CO_2,i} = \nu_s \dot{m}_C$

$$\dot{m}_{O_2} = \nu_f \dot{m}_C = (\nu_s - 1)\dot{m}_C$$

$$\dot{m}_{CO_2,o} = (\nu_f + 1)\dot{m}_C = \nu_s \dot{m}_C$$

Species conservation:

inner zone CO_2: $$\dot{m}_C = \frac{4\pi r^2 \rho D}{(1 + Y_{CO_2}/\nu_s)} \frac{d(Y_{CO_2}/\nu_s)}{dr}$$

BC's: $Y_{CO_2}(r_s) = Y_{CO_2,s}$

$Y_{CO_2}(r_f) = Y_{CO_2,f}$

outter zone CO_2: $$\dot{m}_C = \frac{-4\pi r^2 \rho D}{(1 - Y_{CO_2}/\nu_s)} \frac{d(Y_{CO_2}/\nu_s)}{dr}$$

BC's: $Y_{CO_2}(r_f) = Y_{CO_2,f}$

$Y_{CO_2}(\infty) = 0$

Inert gas (N_2): $$\dot{m}_C = \frac{4\pi r^2 \rho D}{Y_I} \frac{dY_I}{dr}$$

BCs: $Y_I(r_f) = Y_{I,f}$

$Y_I(r \to \infty) = Y_{I,\infty}$

by integrating:

$$\dot{m}_c = 4\pi \left(\frac{r_s r_f}{r_f - r_s}\right) \rho D \ln\left[\frac{1 + Y_{CO_2,f}/\nu_s}{1 + Y_{CO_2,s}/\nu_s}\right]$$

$$\dot{m}_c = -4\pi r_f \rho D \ln\left(1 - Y_{CO_2,f}/\nu_s\right)$$

$$Y_{I,f} = Y_{I,\infty} \exp\left(-\dot{m}_c/(4\pi r_f \rho D)\right)$$

$$Y_{CO_2,f} = 1 - Y_{I,f}$$

Surface kinetics

$$C + CO_2 \Rightarrow 2CO$$

Rate $= k_c [CO_2]$ 1st order w.r.t CO_2

$$\dot{m}_c = 4\pi r_s^2 k_c \frac{MW_c MW_{mix}}{MW_{CO_2}} \frac{P}{R_u T_s} Y_{CO_2,s}$$

$$k_c = 4.016 \times 10^8 \exp\left(-29790/T_s(K)\right)$$

$[m/s]$

if $K_{kin} = 4\pi r_s^2 k_c \frac{MW_c MW_{mix}}{MW_{CO_2}} \frac{P}{R_u T_s}$

$$\dot{m}_c = K_{kin} Y_{CO_2,s}$$

closure

$$\dot{m}_c = 4\pi r_s \rho D \ln\left(1 + B_{CO_2,m}\right)$$

$$B_{CO_2,m} = \frac{2Y_{O_2,\infty} - [(\nu_s - 1)/\nu_s] Y_{CO_2,s}}{(\nu_s - 1) + [(\nu_s - 1)/\nu_s] Y_{CO_2,s}}$$

Particle Burning Times

$$D^2(t) = D_0^2 - K_B t$$

$$K_B = \frac{8 \rho D_{AB}}{\rho_c} \ln(1 + B)$$

B = transfer #

1 film: B_{O_2}

2 film: B_{CO_2}

$$0 = D_0^2 - K_B t_c$$

$$t_c = D_0^2 / K_B$$