$\rho_e \quad V_e \;'' \; K \quad Y_{F,e}$

Assumptions:

1) MW of jet & reservoir fluids are equal (+ I.G. law, const P, const T)
   $\longrightarrow$ uniform fluid ρ throughout flow field
2) molecular transport via Fick's law (binary diffusion)
3) momentum & diffusivity are const & equal. then Schmidt #
   $Sc \equiv \nu / D_{AB}$ where $\nu = \dfrac{\mu}{\rho}$
   is unity
4) only radial diffusion important

Mass conservation:

$$\frac{\partial V_x}{\partial x} + \frac{1}{r}\frac{\partial (V_r r)}{\partial r} = 0$$

Axial momentum conservation:

$$V_x \frac{\partial V_x}{\partial x} + V_r \frac{\partial V_x}{\partial r} = \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_x}{\partial r}\right)$$

$$\nu = \mu/\rho$$

Species cons. for jet fluid (fuel):

$$V_x \frac{\partial Y_F}{\partial x} + V_r \frac{\partial Y_F}{\partial r} = D_{AB}\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial Y_F}{\partial r}\right)$$

and $Y_F = 1 - Y_{ox}$

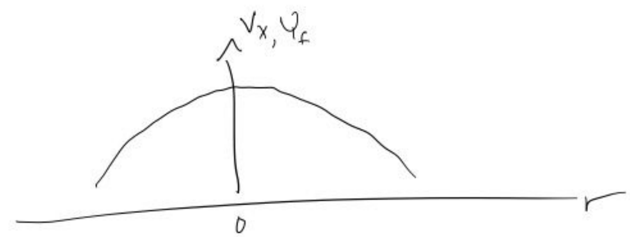
Boundary Conditions

(r=0) centerline

$$V_r(r=0, x) = 0$$

$$\frac{\partial V_x}{\partial r}(0, x) = 0$$

$$\frac{\partial Y_F}{\partial r}(0, x) = 0$$



$(r \to \infty)$ $\quad v_x(\infty, x) = 0 \leftarrow$ stagnant air

$\qquad Y_F(\infty, x) = 0 \leftarrow$ no fuel in air

$(x=0)$ $\quad V_x(r \le R, 0) = V_e$

$\qquad V_x(r > R, 0) = 0$

$\qquad Y_F(r \le R, 0) = 1$

$\qquad Y_F(r > R, 0) = 0$

Solution:

$$\frac{V_x(r,x)}{V_x(0,x)} \quad \left(r/x \right)$$

$$r/x = \text{similarity variable}$$

Axial & radial velocities:

$$v_x = \frac{3}{8\pi} \frac{J_e}{\mu x} \left[1 + \frac{\xi^2}{4} \right]^{-2}$$

$$v_r = \left(\frac{3 J_e}{16 \pi \rho_e} \right)^{1/2} \frac{1}{x} \frac{\xi - \frac{\xi^3}{4}}{\left(1 + \frac{\xi^2}{4}\right)^2}$$

$$J_e = \rho_e V_e^2 \pi R^2$$

$$\xi = \left(\frac{3 \rho_e J_e}{16 \pi} \right)^{1/2} \frac{1}{\mu} \frac{r}{x}$$

dimensionless:

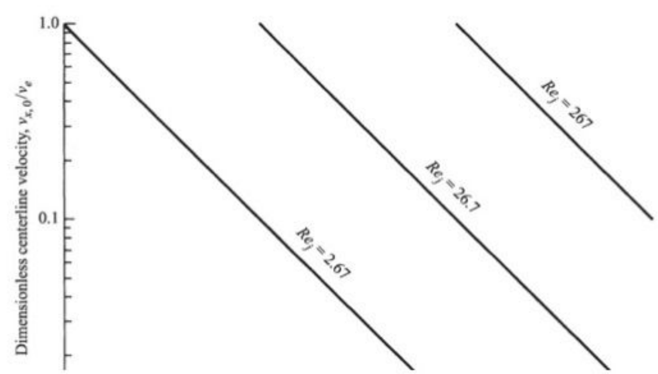
$$\frac{V_x}{V_e} = 0.375 \underbrace{\left(\frac{\rho_e V_e R}{\mu} \right)}_{Re} \left(\frac{x}{R} \right)^{-1} \left[1 + \frac{\xi^2}{4} \right]^{-2}$$

center line:

$r = 0$

$\xi = 0$

$$\frac{V_{x,0}}{V_e} = 0.375 \, Re_j \left(\frac{x}{R} \right)^{-1}$$



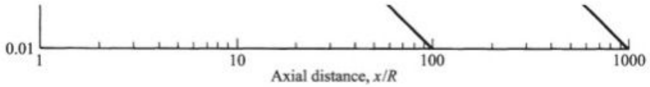


Figure 9.2 Centerline velocity decay for laminar jets.

$r_{1/2}$ (jet half width)

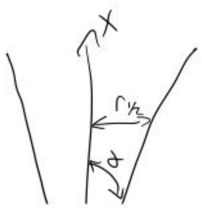
$\frac{V_x}{V_{x,0}} = \frac{1}{2}$ $\rightarrow$ solve for r to get $r_{1/2}$

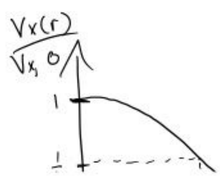
Spreading rate $= r_{1/2}/x$

$$\frac{r_{1/2}}{x} = 2.97 \left(\frac{\mu}{\rho V_e R}\right) = \frac{2.97}{Re_j}$$

Spreading angle (α)

$$\alpha \equiv \tan^{-1}(r_{1/2}/x)$$
$$= \tan^{-1}(2.97/Re_j)$$





- $Sc = 1$ $(\nu = D_{AB}) \rightarrow Y_F$ plays the same role as V_x/V_e

$$Y_F = \frac{3}{8\pi} \frac{Q_F}{D_{AB}\, x} \left[1 + \frac{\xi^2}{4}\right]^{-2}$$

Q_F = vol. flow rate of fuel
$= V_e \pi R^2$

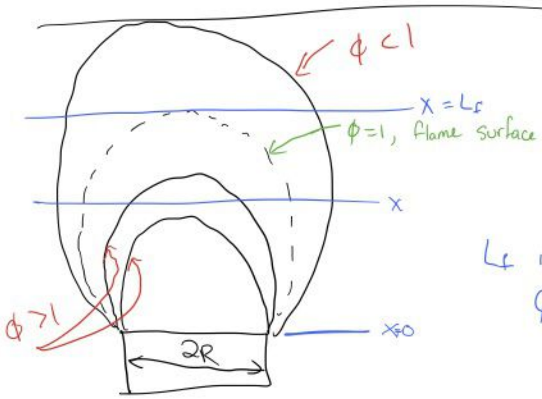
$Sc = 1 \rightarrow \mu/\rho_e = D_{AB}$

$$\hookrightarrow Y_F = \frac{3}{8\pi} \frac{(V_e \pi R^2)}{\mu/\rho_e\,(x)} \left[1 + \frac{\xi^2}{4}\right]^{-2}$$
$$= 0.375\, Re_j \left(\frac{x}{R}\right)^{-1} \left[1 + \frac{\xi^2}{4}\right]^{-2}$$

@ centerline ($r=0$)

$$Y_{F,0} = 0.375\, Re_j \left(\frac{x}{R}\right)^{-1}$$

Solution valid: $\frac{x}{R} \gtrsim 0.375\, Re_j$



L_f is where

$\phi(r=0, x=L_f) = 1$