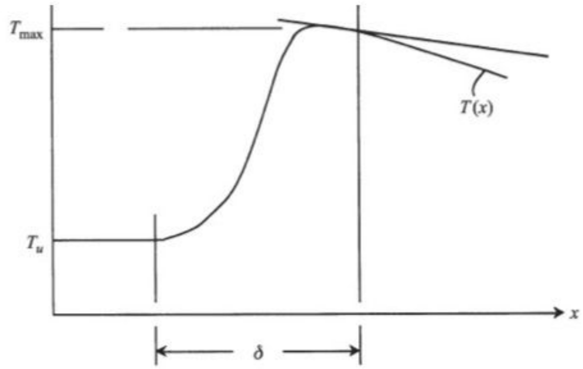
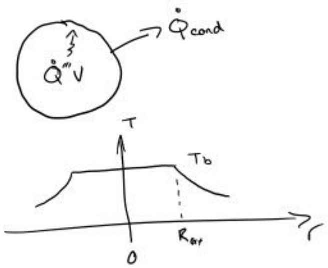


Figure 8.20 Temperature profile through flame with heat losses.

Ignition

Simplified analysis



R_{crit} = minimum radius needed for flame propagation

$\dot{Q}'''V = \dot{Q}_{cond}$

$\dot{Q}''' = -\bar{\dot{m}}'''_F \, \Delta h_c$

$V = \frac{4}{3}\pi R_{crit}^3$

$\dot{Q}_{cond} = -k A \frac{dT}{dr}\Big|_{R_{crit}}$

$A = 4\pi R_{crit}^2$

$+\bar{\dot{m}}'''_F \, \Delta h_c \frac{4}{3}\pi R_{crit}^3 = +k \, 4\pi R_{crit}^2 \frac{dT}{dr}\Big|_{R_{crit}}$

$\frac{\bar{\dot{m}}'''_F \, \Delta h_c}{3} R_{crit} = k \frac{dT}{dr}\Big|_{R_{crit}}$

$R_{crit} = \frac{3k}{\bar{\dot{m}}'''_F \, \Delta h_c} \frac{dT}{dr}\Big|_{R_{crit}}$

$\frac{dT}{dr}\Big|_{R_{crit}} = ?$

$T(R_{crit}) = T_b$

$T(\infty) = T_u$

$\frac{dT}{dr}\Big|_{R_{crit}} = \frac{-(T_b - T_u)}{R_{crit}}$

$R_{crit}^2 = \frac{-3k}{\bar{\dot{m}}'''_F \, \Delta h_c} (T_b - T_u)$

$\alpha = \frac{k}{\rho_u c_p}$

$\Delta h_c = (\nu+1) c_p (T_b - T_u)$

$R_{crit}^2 = \frac{-3\alpha \rho_u}{\bar{\dot{m}}'''_F (\nu+1)}$

$S_L = \left(\frac{-2 \bar{\dot{m}}'''_F (\nu+1) \alpha}{\rho_u} \right)^{1/2}$

$$R_{crit} = \left(\frac{-3\,\alpha\,\rho_u}{\dot{m}_F'''\,(a+1)}\right)^{1/2}$$

$$= \left(\frac{-3(2)\,\alpha\,\alpha\,\rho_u}{2\,\dot{m}_F'''\,(a+1)\,\alpha}\right)^{1/2}$$

$$= \frac{1}{S_L}\left(3(2)\alpha^2\right)^{1/2}$$

$$\boxed{R_{crit} = \frac{\sqrt{6}\,\alpha}{S_L}}$$

$$\delta = \frac{2\alpha}{S_L}$$

$$R_{crit} = \frac{\sqrt{6}}{2}\delta = \boxed{\sqrt{1.5}\,\delta}$$

minimum ignition energy = energy to produce a flame of R_{crit} size

$$E_{ign} = m_{crit}\,c_p\,(T_b - T_u)$$

$$m_{crit} = \rho_b V_{crit} = \rho_b \frac{4}{3}\pi R_{crit}^3$$

$$\rho_B = \frac{P}{R_B T_B} = \frac{P}{(R_u/MW_B)\,T_B}$$

$$m_{crit} = \frac{P\,\frac{4}{3}\pi R_{crit}^3}{(R_u/MW_B)\,T_B}$$

$$m_{crit} = \frac{P}{(R_u/MW_B)\,T_B}\frac{4}{3}\pi\left(\frac{\sqrt{6}\,\alpha}{S_L}\right)^3$$

$$E_{ign} = \frac{P}{(R_u/MW_B)\,T_b}\frac{4}{3}\pi\left(\frac{\sqrt{6}\,\alpha}{S_L}\right)^3 c_p(T_b - T_u)$$

$$E_{ign} = 61.56\,\frac{P\,c_p\,MW_B}{}\left(\frac{T_b - T_u}{T}\right)\left(\frac{\alpha}{S}\right)^3$$

$$\frac{}{R_u}\left(\frac{}{T_b}\right)\left(\frac{}{S_L}\right)$$

P, T, & ϕ dependence

$$\alpha \propto T_u \bar{T}^{0.75} P^{-1}$$

$$S_L \propto \bar{T}^{0.375} T_u T_b^{-n/2} \exp\left(\frac{-E_A}{2R_u T_b}\right) P^{(n-2)/2}$$

$n \approx 2$

$$S_L \propto \bar{T}^{0.375} T_u T_b^{-1} \exp\left(\frac{-E_A}{2R_u T_b}\right)$$

for P: $E_{ign} \propto P\,(P^{-3}) \propto P^{-2}$