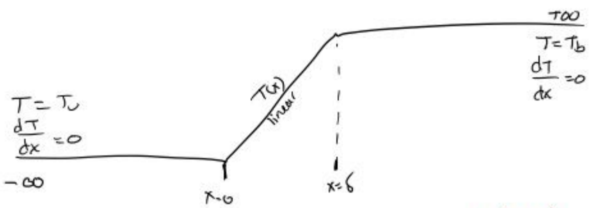
B.C. $T(x \to -\infty) = T_u$

$T(x \to +\infty) = T_b$

$\frac{dT}{dx}(x \to -\infty) = 0$

$\frac{dT}{dx}(x \to +\infty) = 0$



k (Le=1)

$$-\dot m'''_F \Delta h_c = c_p \dot m'' \frac{dT}{dx} - \frac{d}{dx}\left(\rho \cancel{D c_p} \frac{dT}{dx}\right)$$

$$-\dot m'''_F \Delta h_c = c_p \dot m'' \frac{dT}{dx} - \frac{d}{dx}\left(k \frac{dT}{dx}\right)$$

$$-\int_{-\infty}^{\infty} \dot m'''_F \Delta h_c \, dx = \int_{-\infty}^{\infty} c_p \dot m'' \, dT - \int_{-\infty}^{\infty} k \frac{dT}{dx}$$

$$-\Delta h_c \int_{-\infty}^{\infty} \dot m'''_F \, dx = c_p \dot m'' (T_b - T_u) - k\left(\cancel{\frac{dT}{dx}\Big|_{\infty}}^{\,0} - \frac{dT}{dx}\Big|_{-\infty}\right)$$

$$\frac{dT}{dx} = \frac{T_b - T_u}{\delta}$$

$$dx = \frac{\delta}{T_b - T_u} dT$$

$$-\frac{\delta \Delta h_c}{T_b - T_u} \int_{-\infty}^{\infty} \dot m'''_c \, dT = c_p \dot m'' (T_b - T_u)$$

average rxn rate

$$\bar{\dot m}'''_F \equiv \frac{1}{T_b - T_u} \int \dot m'''_o \, dT$$

$$\boxed{\dot m'' c_p (T_b - T_u) = -\Delta h_c \, \delta \, \bar{\dot m}'''_F}$$

need $\dot m''$ & δ → need 1 more eqn.

preheat zone is $-\infty \to \frac{\delta}{2}$

$\dot m'''_F \to 0$ $(x = -\infty \to \delta/2)$

$$T(\delta/2) = \frac{T_b + T_u}{2}$$

$$\frac{dT}{dx}\Big|_{\delta/2} = \frac{T_b - T_u}{\delta}$$

$$\int_{-\infty}^{\delta/2} c_p \dot m'' \, dT - \int_{-\infty}^{\delta/2} \left(k \frac{dT}{dx}\right) = \cancel{\int_{-\infty}^{\delta/2} -\dot m'''_F \Delta h_c \, dx}^{\,0}$$

$$c_p \dot m'' \left(\frac{T_b + T_u}{2} - T_u\right) = k\left(\frac{dT}{dx}\Big|_{\delta/2} - \cancel{\frac{dT}{dx}\Big|_{-\infty}}^{\,0}\right)$$

$$\dot m'' \left(\frac{\cancel{T_b - T_u}}{2}\right) = \frac{k}{c_p}\left(\frac{\cancel{T_b - T_u}}{\delta}\right)$$

$$\boxed{\dot m'' \frac{\delta}{2} = \frac{k}{c_p}}$$

* $$\dot{m}'' = \left[2 \frac{k}{c_p^2} \frac{(-\Delta h_c)}{(T_b - T_u)} \bar{\dot{m}}_F''' \right]^{1/2}$$

* $$\delta = \frac{2k}{c_p \dot{m}''}$$

Flame speed:

for 1-D $S_L \equiv \dot{m}''/\rho_u$

$$S_L = \left[\frac{2k}{\rho_u^2 c_p^2} \frac{(-\Delta h_c)}{(T_b - T_u)} \bar{\dot{m}}_F''' \right]^{1/2}$$

α = thermal diffusivity = $k/\rho_u c_p$

$$S_L = \left[\frac{2\alpha}{\rho_u c_p} \frac{(-\Delta h_c)}{(T_b - T_u)} \bar{\dot{m}}_F''' \right]^{1/2}$$

$$\Delta h_c = (a+1) c_p (T_b - T_u)$$

$$\boxed{S_L = \left[\frac{2\alpha}{\rho_u} (-(a+1)) \bar{\dot{m}}_F''' \right]^{1/2}}$$

1 kg fuel + a kg oxidizer

$$\delta = \frac{2k}{c_p}\left(\frac{1}{\dot{m}''}\right) = \overbrace{\frac{2k}{c_p}\left(\frac{1}{S_L \rho_u}\right)}^{\alpha}$$

$$\boxed{\delta = \frac{2\alpha}{S_L}}$$

Detailed Analysis:

continuity: $$\frac{d\dot{m}''}{dx} = 0$$

Species: $$\dot{m}'' \frac{dY_i}{dx} + \frac{d}{dx}\left(\rho Y_i v_{i,diff}\right) = \dot{\omega}_i MW_i$$

energy: $$\dot{m}'' c_p \left(\frac{dT}{dx}\right) + \frac{d}{dx}\left(-k \frac{dT}{dx}\right) + \sum_{i=1}^{N} \rho Y_i v_{i,diff} c_{p_i} \frac{dT}{dx} = -\sum_{i=1}^{N} h_i \dot{\omega}_i MW_i$$

B.C.

$$T(x \to -\infty) = T_u$$

$$\left.\frac{dT}{dx}\right| = 0$$

$dx\,|_{+\infty}$

$Y_i\,(x \to -\infty) = Y_{i,u}$

$$\left.\frac{dY_i}{dx}\right|_{\infty} = 0$$

$T(x_1) = T_1$

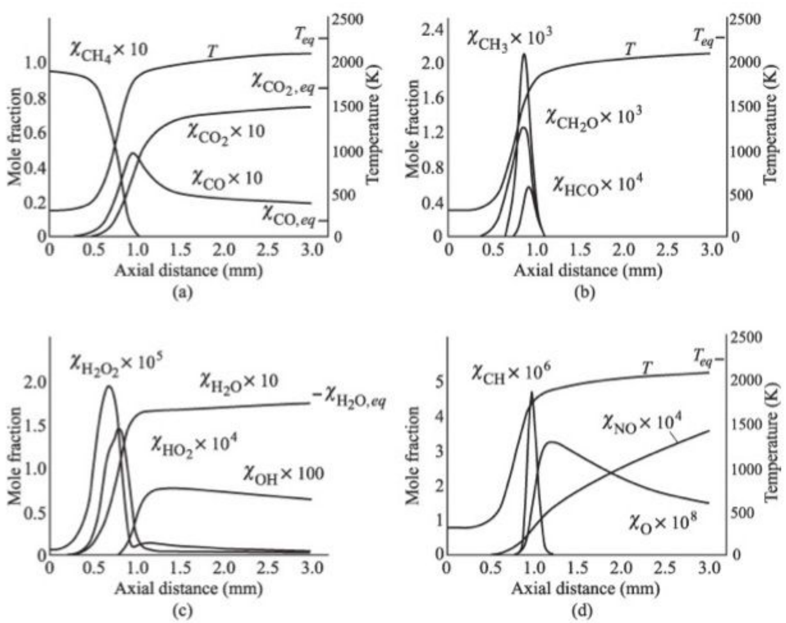


Figure 8.10 Calculated species mole-fraction and temperature profiles for laminar, stoichiometric, CH_4–air premixed flame. (a) T, χ_{CH_4}, χ_{CO}, and χ_{CO_2}; (b) T, χ_{CH_3}, χ_{CH_2O}, and χ_{HCO}; (c) χ_{H_2O}, χ_{OH}, $\chi_{H_2O_2}$, and χ_{HO_2} ; (d) T, χ_{CH}, χ_{O}, and χ_{NO}.

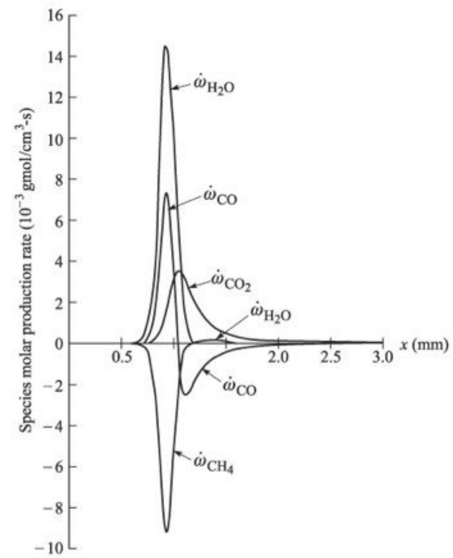


Figure 8.11 Calculated volumetric species production rate profiles for laminar, stoichiometric, CH_4–air premixed flame. Corresponds to the same conditions as in Fig. 8.10.

Factors that influence flame v_s δ

Temperature:

$$\alpha \equiv \frac{k}{c_p \rho_u} \propto \underbrace{\frac{T_u}{P}}_{\rho_u} \underbrace{\bar{T}^{0.75}}_{k/c_p}$$

$$\dot{m}_i''' = A \exp(-E/R_uT)[x_i]^n$$

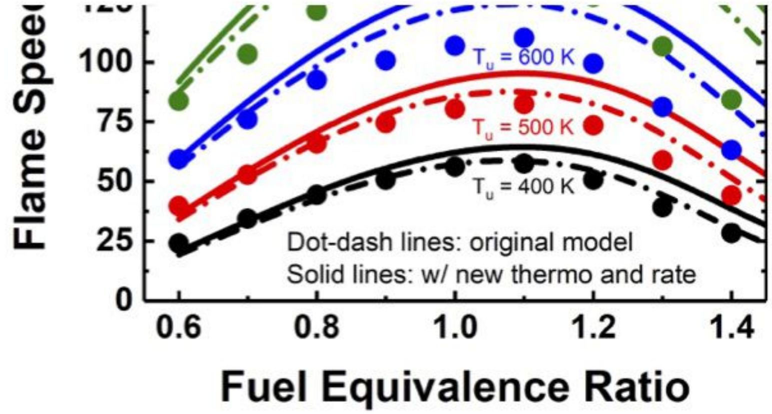
$$\frac{\quad}{\rho_u} = \frac{\quad}{\rho_u}$$

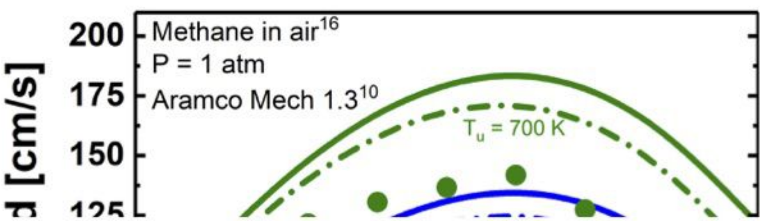
$$\propto \left(\frac{T_u}{P}\right)\left(\frac{P}{T_b}\right)^n \exp\left(-E_A/R_u T_b\right)$$

$$\Rightarrow S_L \propto \bar{T}^{0.375} T_u T_b^{-n/2} \exp\left(-E_A/2R_u T_b\right) P^{(n-2)/2}$$

$$\delta \propto \bar{T}^{0.375} T_b^{n/2} \exp\left(+E_A/2R_u T_b\right) P^{-n/2}$$

	A	B	C
T_u	300 K	600 K	300 K
T_b	2000 K	~2300 K	1700 K
S_L/S_{L_A}	1	3.64	0.46
δ/δ_A	1	0.65	1.95





Pressure

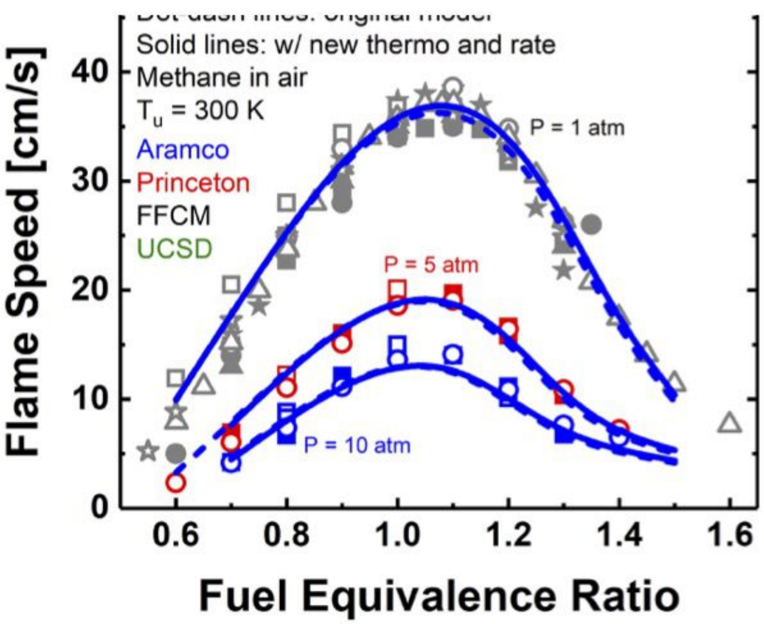
$$S_L \propto P^{(n-2)/2}$$

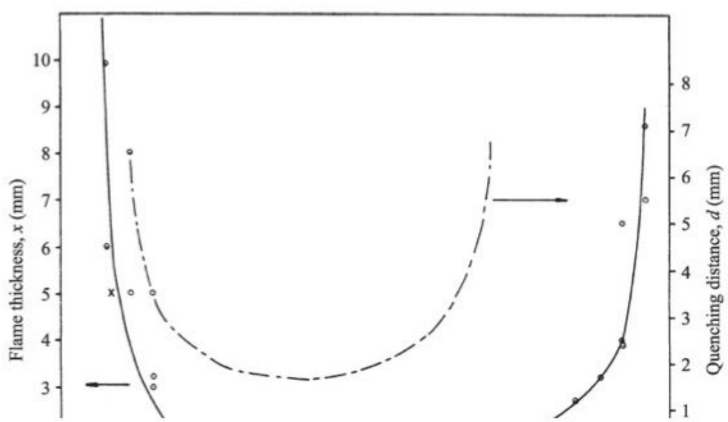
$P \uparrow \quad S_L \downarrow$ for $n < 2$

$P \uparrow \quad S_L$ const for $n = 2$

$\forall$ order < 2

$$\delta \propto P^{-n/2}$$





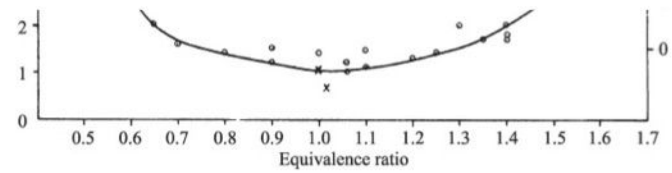


Figure 8.16 Flame thickness for laminar methane–air flames at atmospheric pressure. Also shown is the quenching distance.

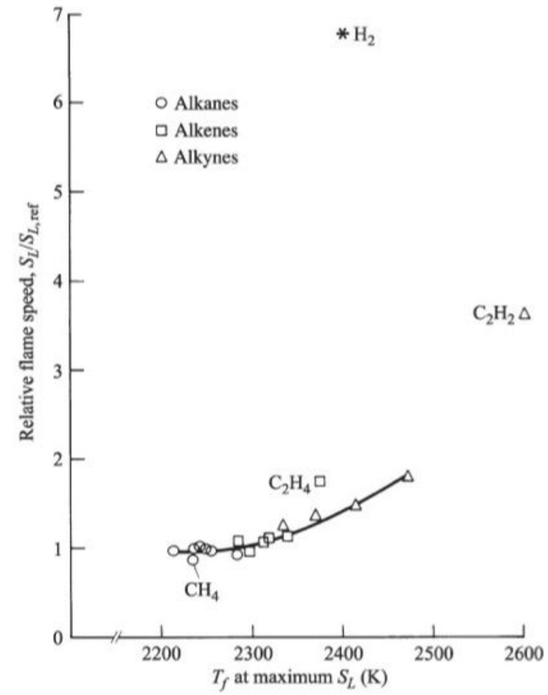


Figure 8.17 Relative flame speeds for C_1–C_6 hydrocarbon fuels. The reference flame speed is based on propane using the tube method [21].

Metghalchi & Keck

$$S_L = S_{L,ref}\left(\frac{T_u}{T_{u,ref}}\right)^{\gamma}\left(\frac{P}{P_{ref}}\right)^{\beta}(1-2.1\,Y_{dil})$$

$T_u > 350K$

$T_{u,ref} = 298K$

$P_{ref} = 1atm$

$$S_{L,ref} = B_m + B_2(\phi - \phi_m)^2$$

$\gamma = 2.18 - 0.8(\phi - 1)$

$\beta = -0.16 + 0.22(\phi - 1)$

Y_{dil} = mass frac. of dilluent