Figure 8.5 (a) Adiabatic flat-flame burner. (b) Nonadiabatic flat-flame burner.

Simplified Flame Analysis

Assumptions:

1) 1-D, constant A, steady flow
2) KE, PE, viscous shear work & thermal radiation all ignored
3) Small P differences across the flame ignored (P = const.)
4) heat & mass diffusion are governed by Fourier's & Fick's laws:
   Binary Diffusion assumed.

   Fouriers: $\dot{Q}''_x = -k \frac{dT}{dx}$

   Ficks: $\dot{m}''_i = \dot{m}'' Y_i - \rho D \frac{dY_i}{dx}$
5) Lewis #, Le = 1

$$1 = \frac{\alpha}{} = \frac{k}{} = 1$$

$$Le = \frac{}{D} \quad \rho c_p D$$

$$\frac{k}{c_p} = \rho D$$

$$Le = \frac{Sc}{Pr} \qquad Sc = \text{schmidt \#} \quad Pr = \text{Prandtl \#}$$

$$Sc = \frac{\text{viscous diffusion rate}}{\text{molecular diffusion rate}}$$

$Sc \ll 1 \rightarrow$ mol. diff. dominates

$Sc \gg 1 \rightarrow$ convection dominates

$$Pr = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}$$

mercury: $Pr \cong 0.015$
(conduction dominant
– good thermometer!

engine oil: $Pr = 100 - 40000$
(convection dominant

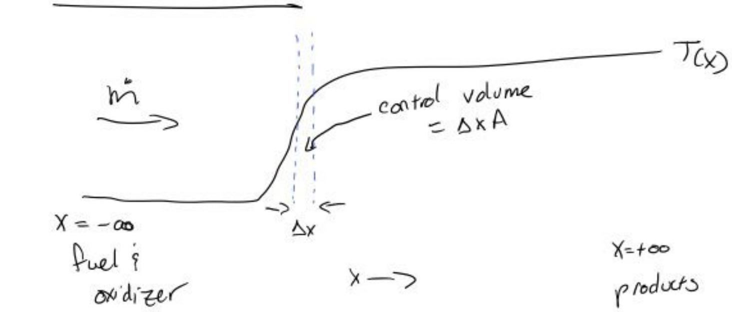
6) Cp does not depend on T or composition

mixture composition

7) 1 step global chemistry

8) $\phi = 1$ or less

Conservation laws:



Mass conservation:

$$\frac{d(\rho v_x)}{dx} = 0$$

$$\dot{m}'' = \rho v_x = \text{const.}$$

Continuity: $\rho_{in} V_{in} \cancel{A} = \rho_{out} V_{out} \cancel{A}$

Species conservation:

$$\underbrace{\dot{m}_i'''}_{\text{source/sink due to reaction}} = \underbrace{\frac{d}{dx}(\dot{m}_i'')}_{\text{change in flux across differential } V}$$

$$\dot{m}_i'' = \dot{m}'' Y_i - \rho D \frac{dY_i}{dx}$$

$$\frac{d}{dx}\left(\dot{m}'' Y_i - \rho D \frac{dY_i}{dx}\right) = \dot{m}_i''' \leftarrow$$

Single step rxn

1 kg fuel + a kg oxidizer
$\rightarrow$ (1 + a) kg products

$$\dot{m}_F''' = \frac{1}{a}\dot{m}_{ox}''' = \frac{-1}{1+a}\dot{m}_{pr}'''$$

for fuel:

$$\frac{d}{dx}\left(\dot{m}'' Y_F - \rho D \frac{dY_F}{dx}\right) = \dot{m}_F'''$$

$$\dot{m}'' \frac{dY_F}{dx} - \frac{d}{dx}\left(\rho D \frac{dY_F}{dx}\right) = \dot{m}_F'''$$

for oxidizer:

$$\dot{m}'' \frac{dY_{ox}}{dx} - \frac{d}{dx}\left(\rho D \frac{dY_{ox}}{dx}\right) = a\,\dot{m}_F'''$$

for products:

$$\dot{m}'' \frac{dY_{Pr}}{dx} - \frac{d}{dx}\left(\rho D \frac{dY_{Pr}}{dx}\right) = -(1+a)\,\dot{m}_F'''$$

Energy conservation:

1-D system:

1st law:

$$\dot{Q} - \cancel{\dot{W}} = (\dot{m}h)_{in} - (\dot{m}h)_{out} + \cancel{\left(\dot{m}\frac{V_x^2}{2}\right)_{in} - \left(\dot{m}\frac{V_x^2}{2}\right)_{out}} + \cancel{PE}$$

$$\left(\dot{Q}''_{x+\Delta x} - \dot{Q}''_x\right)\cancel{A} = \dot{m}''\cancel{A}\left[h_x - h_{x+\Delta x}\right]$$

$\Delta x \to 0$

$$-\frac{d\dot{Q}''_x}{dx} = \dot{m}''\frac{dh}{dx}$$

what is Q?

$$\dot{Q}'' = -k\frac{dT}{dx} + \underbrace{\sum \dot{m}''_{i,diff} h_i}_{\text{Species diffusion E contribution}}$$

$$\dot{m}''_{i,diff} = -\rho D \frac{dY_i}{dx}$$

$$\dot{Q}'' = -k\frac{dT}{dx} - \rho D \sum h_i \frac{dY_i}{dx}$$

$$\frac{d\sum h_i Y_i}{dx} = \sum h_i \frac{dY_i}{dx} + \sum Y_i \frac{dh_i}{dx}$$

$$\dot{Q}'' = -k\frac{dT}{dx} - \rho D \frac{d\sum h_i Y_i}{dx} + \rho D \sum Y_i \frac{dh_i}{dx}$$

$$h = \sum h_i Y_i$$

$$\frac{dh_i}{dx} = c_{p_i}\frac{dT}{dx}$$

$$\dot{Q}'' = -k\frac{dT}{dx} - \rho D\frac{dh}{dx} + \rho D \overset{c_p}{\boxed{\sum Y_i c_{p_i}}}\frac{dT}{dx}$$

$$= -k\frac{dT}{dx} - \rho D\frac{dh}{dx} + \rho D c_p \frac{dT}{dx}$$

Assumpt # 5: $Le = 1$

$$k = \rho D c_p$$

$$\dot{Q}'' = -\rho D \frac{dh}{dx} + (\cancel{\rho D c_p - k})\frac{dT}{dx}$$

$$\dot{Q}'' = -\rho D \frac{dh}{dx}$$

$$\frac{-d\dot{Q}_x''}{dx} = \dot{m}'' \frac{dh}{dx}$$

$$+\frac{d}{dx}\left(+\rho D \frac{dh}{dx}\right) = \dot{m}'' \frac{dh}{dx}$$

$$\Rightarrow \frac{d}{dx}\left(\rho D \frac{dh}{dx}\right) = \dot{m}'' \frac{dh}{dx}$$

$$dh = \sum dY_i h_{f,i}^\circ + \sum Y_i c_{p_i} dT$$

$$= \sum dY_i h_{fi}^\circ + c_p dT$$

$$\frac{d}{dx}\left(\rho D \sum h_{fi}^\circ \frac{dY_i}{dx} + \rho D c_p \frac{dT}{dx}\right) =$$

$$\dot{m}''\left(\sum h_f^\circ \frac{dY_i}{dx} + c_p \frac{dT}{dx}\right)$$

$$\frac{d}{dx}\left(\rho D \sum h_{fi}^\circ \frac{dY_i}{dx}\right) - \dot{m}'' \sum h_{fi}^\circ \frac{dY_i}{dx} =$$

$$\dot{m}'' c_p \frac{dT}{dx} - \frac{d}{dx}\left(\rho D c_p \frac{dT}{dx}\right)$$

LHS: $$\frac{d}{dx}\left(\rho D \sum h_{fi}^\circ \frac{dY_i}{dx} - \sum h_{fi}^\circ \dot{m}'' Y_i\right) =$$

$$\frac{d}{dx}\left(\sum h_{fi}^\circ \left(\rho D \frac{dY_i}{dx} - \dot{m}'' Y_i\right)\right) =$$

$$-\frac{d}{dx}\left(\sum h_{fi}^\circ \dot{m}_i''\right) = -\sum h_{f,i}^\circ \frac{d\dot{m}_i''}{dx}$$

$$= -\sum h_{fi}^\circ \dot{m}_i'''$$

$$\boxed{-\sum h_{f,i}^\circ \dot{m}_i''' = \dot{m}'' c_p \frac{dT}{dx} - \frac{d}{dx}\left(\rho D c_p \frac{dT}{dx}\right)}$$

Shvab – Zeldovich energy eqn.

$$\sum h_{f,i}^\circ \dot{m}_i''' = h_{fF}^\circ \dot{m}_F''' + h_{f,ox}^\circ \dot{m}_{ox}''' + h_{fpr}^\circ \dot{m}_{pr}'''$$

$$= h_{f,F}^\circ \dot{m}_F''' + a h_{f,ox}^\circ \dot{m}_F''' + -(1+a) h_{f,pr}^\circ \dot{m}_F'''$$

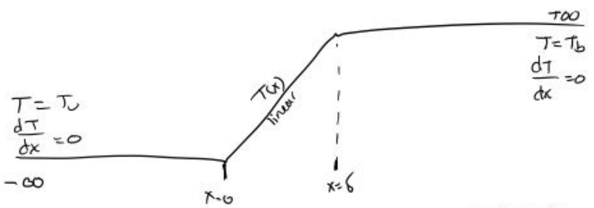
$$= \dot{m}_F'''\left(h_{f,F}^\circ + a h_{f,ox}^\circ - (1+a) h_{f,pr}^\circ\right)$$

$$= \dot{m}_F''' \Delta h_c$$

$$\boxed{-\dot{m}_F''' \Delta h_c = \dot{m}'' c_p \frac{dT}{dx} - \frac{d}{dx}\left(\rho D c_p \frac{dT}{dx}\right)}$$

Solving all 3 eqns:

B.C.
$$T(x \to -\infty) = T_u$$
$$T(x \to +\infty) = T_b$$
$$\frac{dT}{dx}(x \to -\infty) = 0$$
$$\frac{dT}{dx}(x \to +\infty) = 0$$



$$-\dot{m}'''_c \Delta h_c = c_p \dot{m}'' \frac{dT}{dx} - \frac{d}{dx}\left(\overset{k\ (Le=1)}{\cancel{\rho D}}\, c_p \frac{dT}{dx}\right)$$

$$-\dot{m}'''_F \Delta h_c = c_p \dot{m}'' \frac{dT}{dx} - \frac{d}{dx}\left(k \frac{dT}{dx}\right)$$

$$-\int_{-\infty}^{\infty} \dot{m}'''_F \Delta h_c \, dx = \int_{-\infty}^{\infty} c_p \dot{m}'' \, dT - \int_{-\infty}^{\infty} k \frac{dT}{dx}$$

$$-\Delta h_c \int_{-\infty}^{\infty} \dot{m}'''_F \, dx = c_p \dot{m}''(T_b - T_u) - k \cancelto{0}{\left(\frac{dT}{dx}\Big|_{\infty} - \frac{dT}{dx}\Big|_{-\infty}\right)}$$

$$\frac{dT}{dx} = \frac{T_b - T_u}{\delta}$$

$$dx = \frac{\delta}{T_b - T_u} \, dT$$

$$-\frac{\delta \Delta h_c}{T_b - T_u} \int_{-\infty}^{\infty} \dot{m}'''_c \, dT = c_p \dot{m}''(T_b - T_u)$$

average rxn rate

$$\bar{\dot{m}}'''_F \equiv \frac{1}{T_b - T_u} \int \dot{m}'''_c \, dT$$

$$\boxed{\dot{m}'' c_p (T_b - T_u) = -\Delta h_c \, \delta \, \bar{\dot{m}}'''_F}$$

need $\dot{m}''$ & δ → need 1 more eqn.

preheat zone is $-\infty \to \frac{\delta}{2}$

$\dot{m}'''_F \to 0$ $(x = -\infty \to \delta/2)$

$$T(\delta/2) = \frac{T_b + T_u}{2}$$

$$\frac{dT}{dx}\Big|_{\delta/2} = \frac{T_b - T_u}{\delta}$$