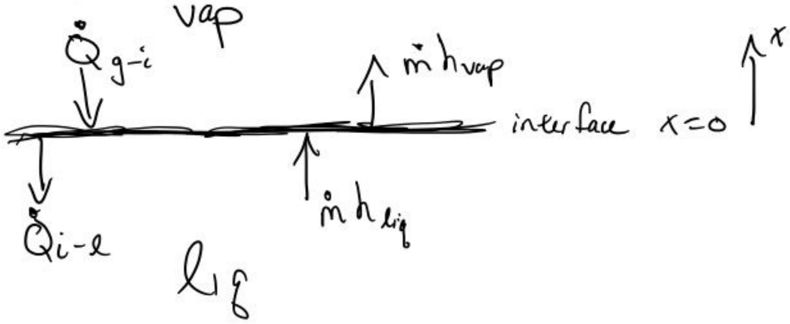
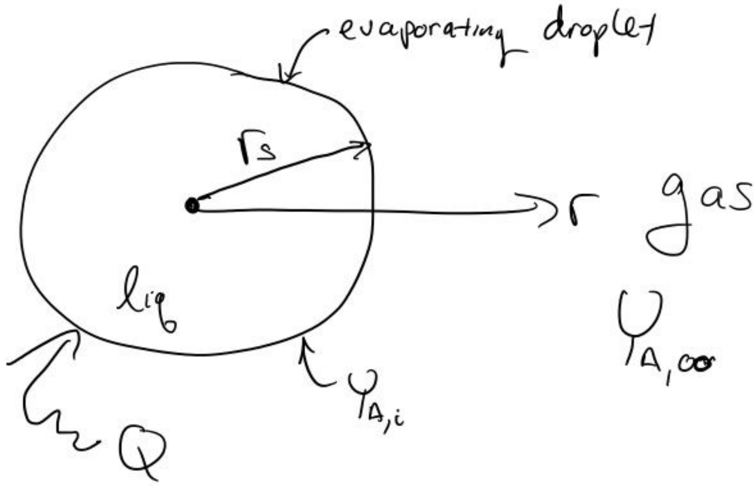
$T_{liq,i}(x=0^-) = T_{vap,i}(x=0^+) = T(0)$



energy balance: $\dot{Q}_{g-i} - \dot{Q}_{i-l} = \dot{m}(h_{vap} - h_{liq})$

$$\dot{Q}_{net} = \dot{m}h_{fg}$$

Droplet evaporation



- radius is our only coordinate variable (r)
- r_s changes w/ time
- $r \to \infty$ $Y_A = Y_{A,\infty}$
- heat from ambient gas supplies the

heat for evaporation"

- ignore chemistry

eqn for system:

- mass cons. for droplet
- mass cons. for surroundings
- energy cons. for droplet
- energy cons. for surroundings
- mass cons. across the boundary

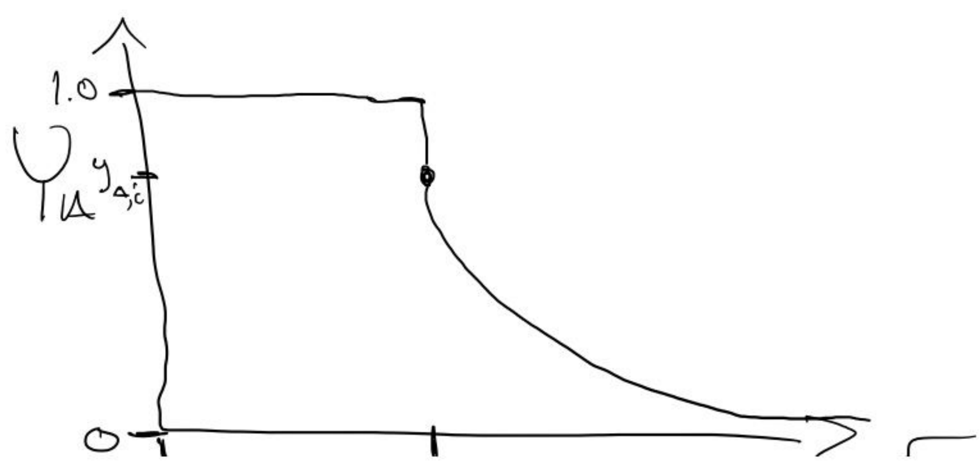
Assumptions:

#1: Quasi-steady state evaporation

#2: Droplet T is uniform

#3: $X_{A,i} = \frac{P_{sat}(T_{liq,i})}{P}$ (vapor-liq. equil.)

#4: thermophysical properties are constant



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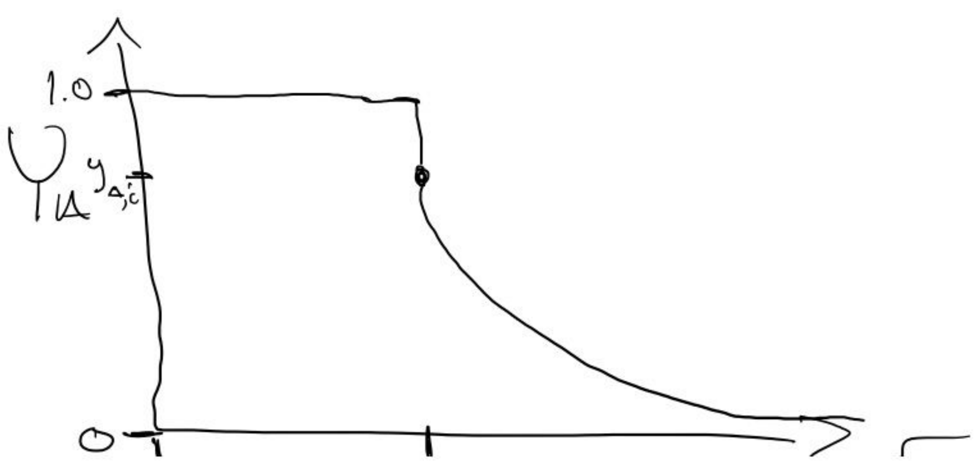

Apply BC $r = r_s$, $Y_A = Y_{AS}$

$$\ln\left(1 - Y_{A,S}\right) + \frac{\dot{m}}{4\pi r_s \rho D_{AB}} = C$$

$$\Rightarrow \quad \frac{\dot{m}}{4\pi r \rho D_{AB}} = -\ln\left(1 - Y_A\right) + \ln\left(1 - Y_{A,s}\right) + \frac{\dot{m}}{4\pi r_s \rho D_{AB}}$$

$$\frac{-\dot{m}}{4\pi \rho D_{AB}}\left(\frac{1}{r_s} - \frac{1}{r}\right) = \ln\left(\frac{1 - Y_{A,s}}{1 - Y_A}\right)$$

$$\frac{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r_s}\right)}{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r}\right)} = \frac{1 - Y_{A,s}}{1 - Y_A}$$

$$\Rightarrow \quad Y_A(r) = 1 - \left(1 - Y_{A,s}\right)\frac{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r}\right)}{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r_s}\right)}$$

Apply BC $\rightarrow$ $r \rightarrow \infty$ $Y_A \rightarrow Y_{A,\infty}$

$$Y_{A,\infty} = 1 - \left(1 - Y_{A,s}\right)\frac{\cancel{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} \infty}\right)}^{\,1}}{\exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r_s}\right)}$$

$$\frac{\left(1 - Y_{A,\infty}\right)}{\left(1 - Y_{A,s}\right)} = \exp\left(\frac{-\dot{m}}{4\pi \rho D_{AB} r_s}\right)$$

$$\dot{m} = 4\pi \rho D_{AB} r_s \ln\left(\frac{1 - Y_{A,\infty}}{1 - Y_{A,s}}\right)$$

transfer #: B

$$1 + B_Y \equiv \frac{1 - Y_{A,\infty}}{1 - Y_{A,s}}$$

$$B_Y \rightarrow \frac{Y_{A,s} - Y_{A,\infty}}{1 - Y_{A,s}}$$

$$\dot{m} = 4\pi r_s \rho D_{AB} \ln(1 + B_Y)$$

$$B_Y \rightarrow 0 \qquad \dot{m} \rightarrow 0$$

$$B_Y \uparrow \qquad \dot{m} \uparrow$$

Droplet mass conservation:

$$\frac{d\, m_{drop}}{\partial t} = -\dot{m}$$

$$m_{drop} = \rho_{liq} V = \rho_{liq} \frac{\pi D^3}{6}$$

$$= \rho_{liq} \pi \frac{(8 r_s^3)}{6}$$

$$\frac{d\left(\frac{\cancel{4}}{3} \cancel{\pi} r_s^3 \rho_\ell\right)}{dt} = -\dot{m} = -\cancel{4}\cancel{\pi} r_s \rho D_{AB} \ln(1+B_Y)$$

$$\frac{1}{3} \rho_\ell \frac{d r_s^3}{\partial t} = -r_s \rho D_{AB} \ln(1 + B_Y)$$

$$\cancel{3}\, r_s^2 \frac{\rho_\ell}{\cancel{3}} \frac{dr_s}{dt} = -r_s \rho\, D_{AB} \ln(1+B_y)$$

$$\frac{dr_s}{\partial t} = \frac{-\rho}{\rho_\ell} \frac{D_{AB}}{r_s} \ln(1+B_y)$$

$$\frac{dD}{\partial t} = \frac{-4\rho}{\rho_\ell} \frac{D_{AB}}{D} \ln(1+B_y)$$

$$\frac{dD^2}{\partial t} = \frac{-8\rho}{\rho_\ell} D_{AB} \ln(1+B_Y)$$

evaporation const. K

$$K = \frac{8\rho}{\rho_\ell} D_{AB} \ln(1+B_Y)$$

$$\frac{\partial D^2}{\partial t} = -K$$

t_d = time to completely evaporate droplet

$$\int_{D_0^2}^{0} \partial D^2 = \int_0^{t_d} -K$$

$$t_d = D_0^2 / K$$

or $D^2(t) = D_0^2 - Kt$ $\boxed{D^2 \text{ law}}$

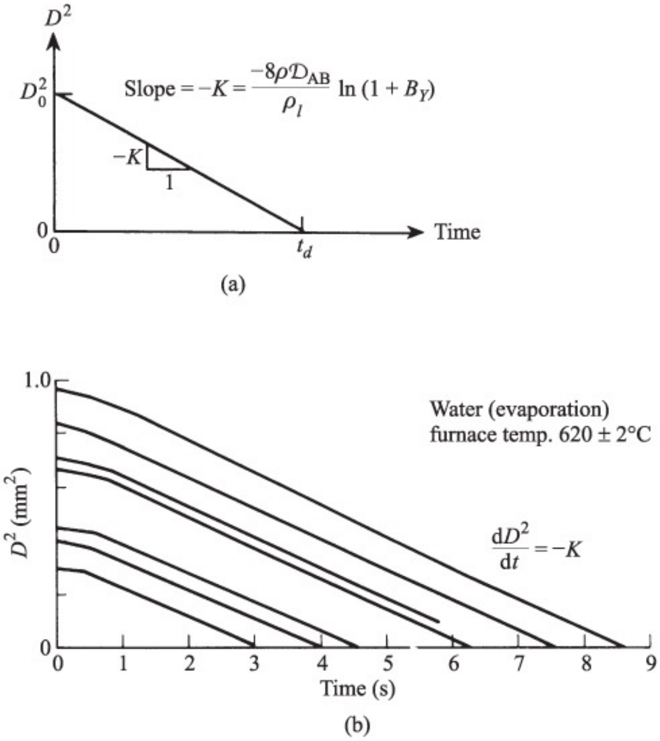

Figure 3.7 The D^2 law for droplet evaporation. (a) Simplified analysis. (b) Experimental data from Ref. [8] for water droplets with $T_\infty = 620°C$.